Automatic Control (1)

By

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Lecture (3) – Part (1)

Block Diagram Fundamentals
&
Reduction Techniques
&
Signal-flow Graphs
&
State-Space Representation

Superposition of Multiple Inputs

Step 1: Set all inputs except one equal to zero.

Step 2: Transform the block diagram to canonical form, using the transformations of Section 7.5.

Step 3: Calculate the response due to the chosen input acting alone.

Step 4: Repeat Steps 1 to 3 for each of the remaining inputs.

Step 5: Algebraically add all of the responses (outputs) determined in Steps 1 to 4. This sum is the total output of the system with all inputs acting simultaneously.

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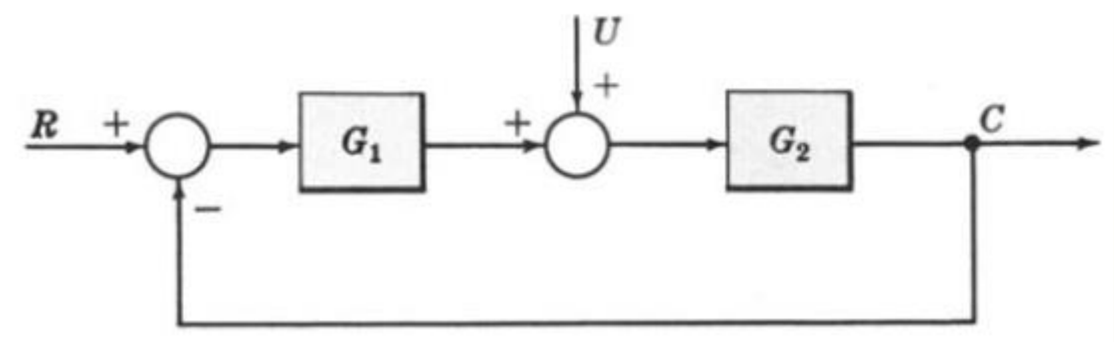
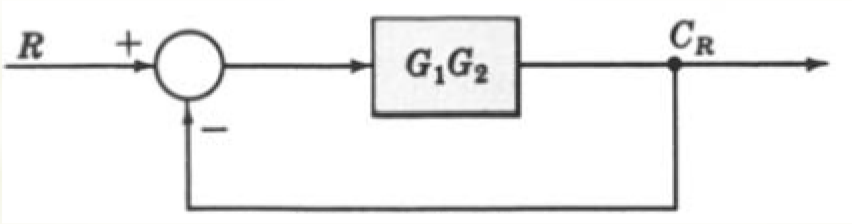
Example-15: Multiple Input System. Determine the output C due to inputs R and U using the Superposition Method.



Step 1: Put $U \equiv 0$.
Step 2: The system reduces to



Step 3: the output C_R due to input R is $C_R = [G_1G_2/(1 + G_1G_2)]R$

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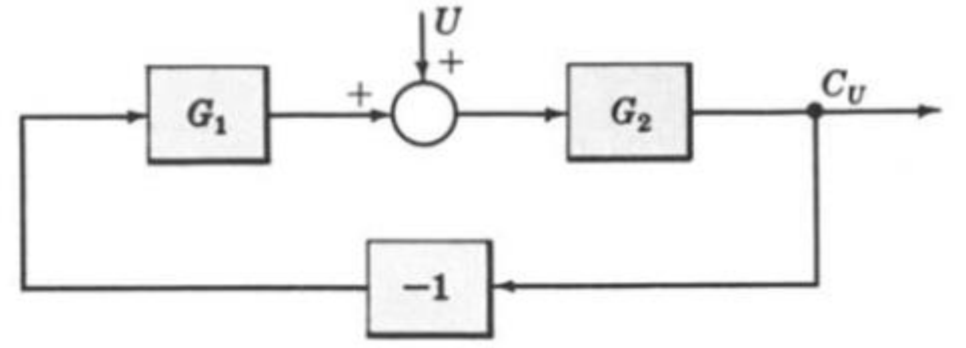
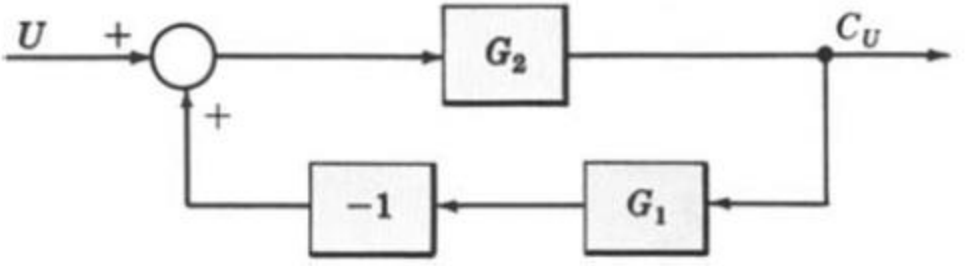
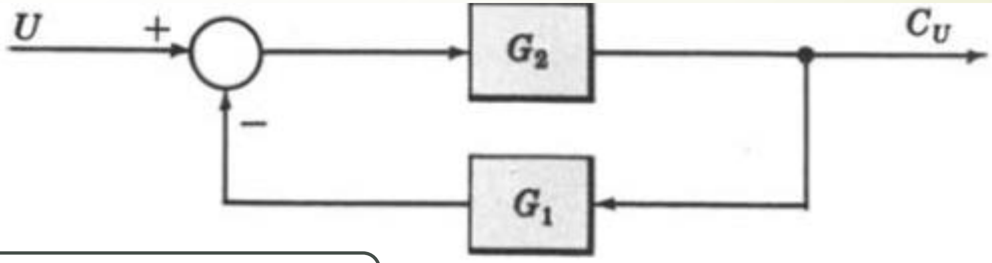


Step 4a: Put $R = 0$.
Step 4b: Put -1 into a block, representing the negative feedback effect:

Rearrange the block diagram:



Let the -1 block be absorbed into the summing point:



Step 4c: the output C_U due to input U is $C_U = [G_2/(1 + G_1G_2)]U$.

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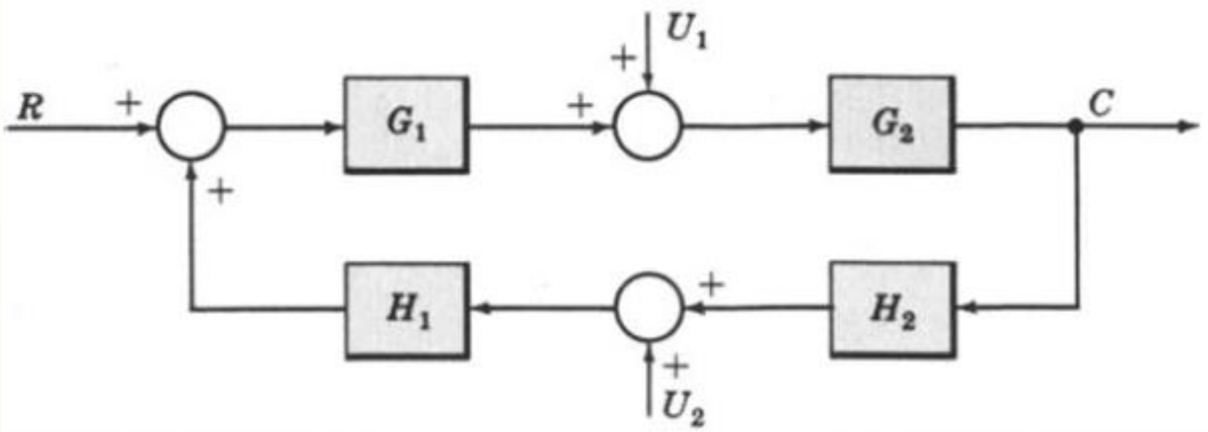
Example-15: Continue

Step 5: The total output is $C = C_R + C_U$

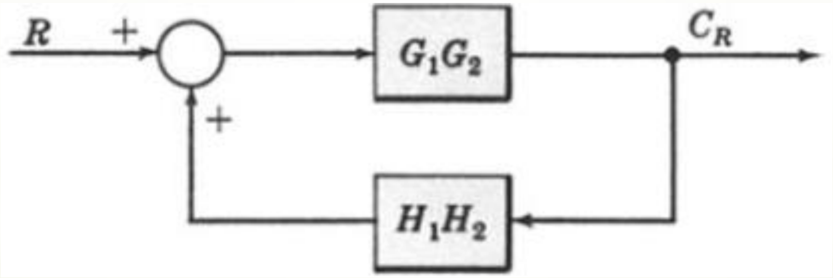
$$= \left[\frac{G_1G_2}{1 + G_1G_2}\right]R + \left[\frac{G_2}{1 + G_1G_2}\right]U$$

$$= \left[\frac{G_2}{1 + G_1G_2}\right][G_1R + U]$$

Example-16: **Multiple-Input System**. Determine the output C due to inputs R, U_1 and U_2 using the Superposition Method.



Let $U_1 = U_2 = 0$.



$$C_R = [G_1G_2/(1 - G_1G_2H_1H_2)]R$$

where C_R is the output due to R acting alone.

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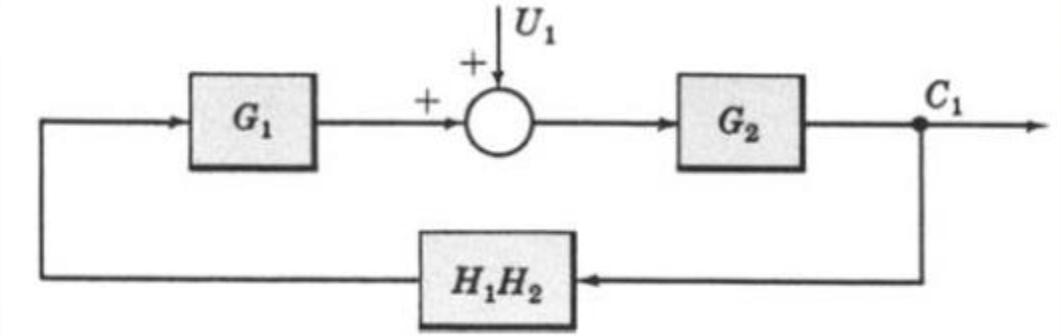
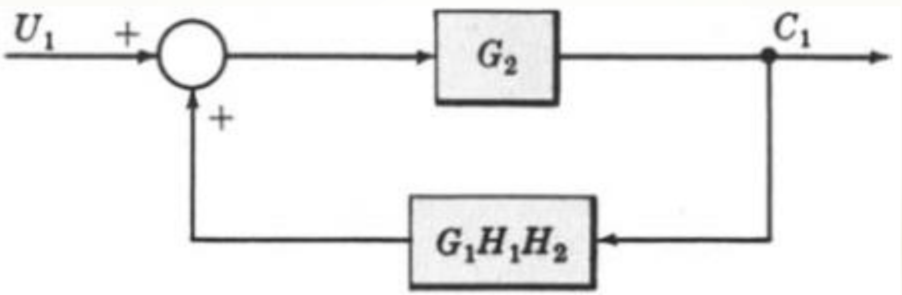
Example-16: Continue

Now let $R = U_2 = 0$.



Rearranging the blocks, we get

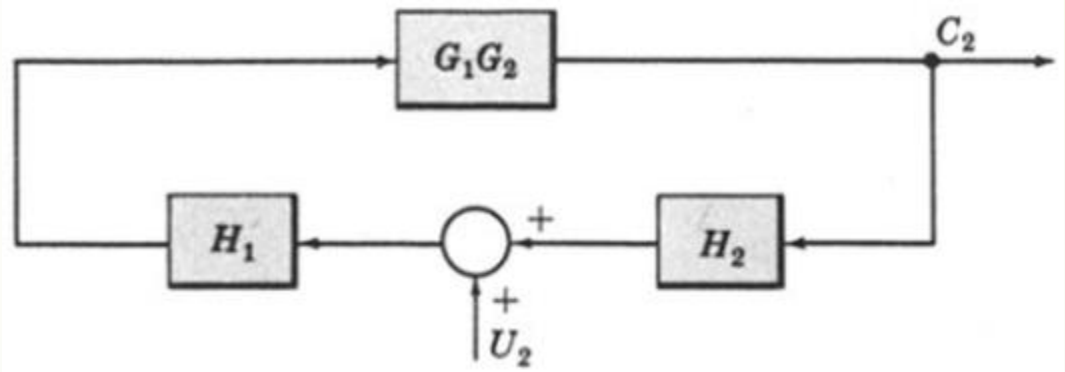


$$C_1 = [G_2/(1 - G_1G_2H_1H_2)]U_1$$

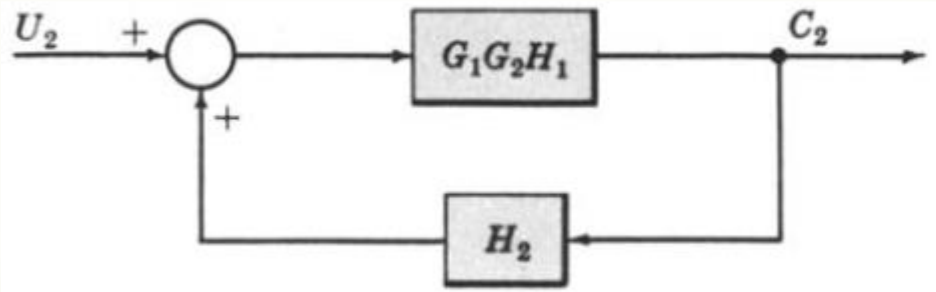
where C_1 is the response due to U_1 acting alone.

Example-16: Continue

Finally, let $R = U_1 = 0$.



Rearranging the blocks, we get



$$C_2 = [G_1G_2H_1/(1 - G_1G_2H_1H_2)]U_2$$

where C_2 is the response due to U_2 acting alone.

By superposition, the total output is

$$C = C_R + C_1 + C_2 = \frac{G_1G_2R + G_2U_1 + G_1G_2H_1U_2}{1 - G_1G_2H_1H_2}$$

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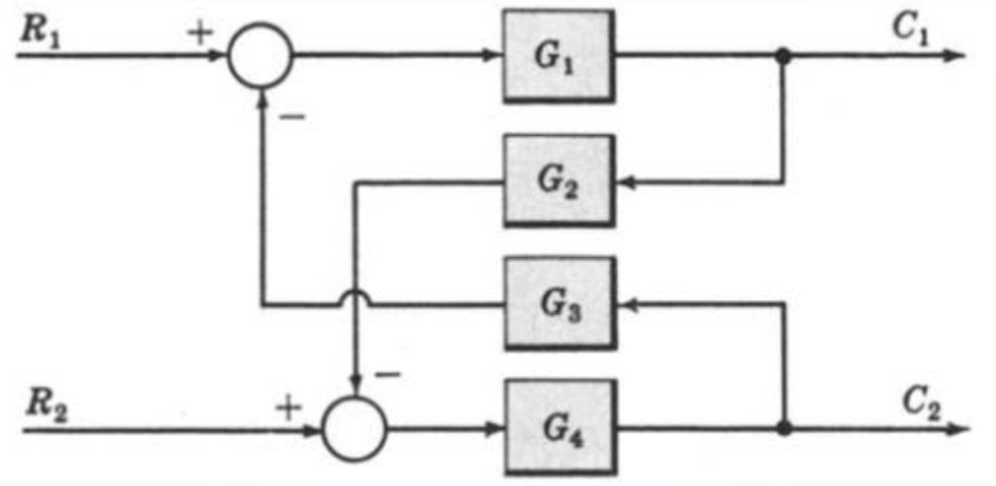
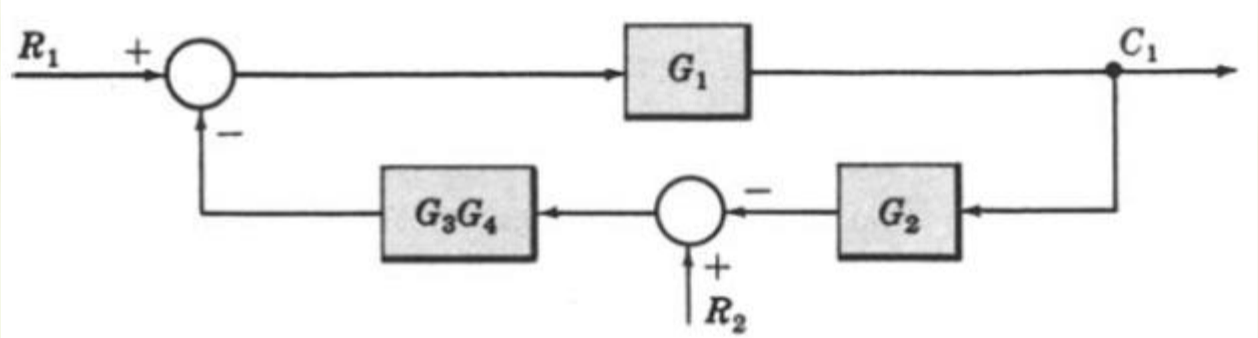
Example-17: Multi-Input Multi-Output System.
Determine C_1 and C_2 due to R_1 and R_2.



First ignoring the output C_2.



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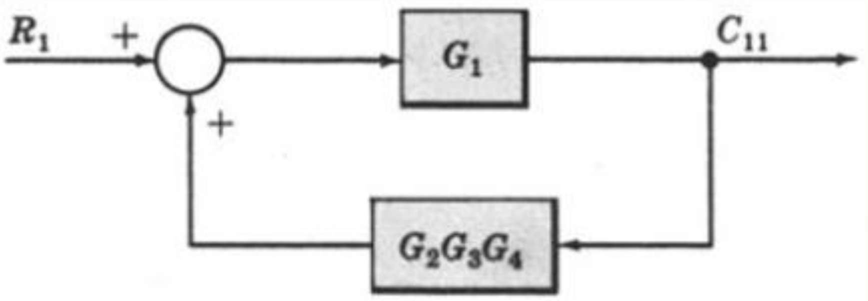
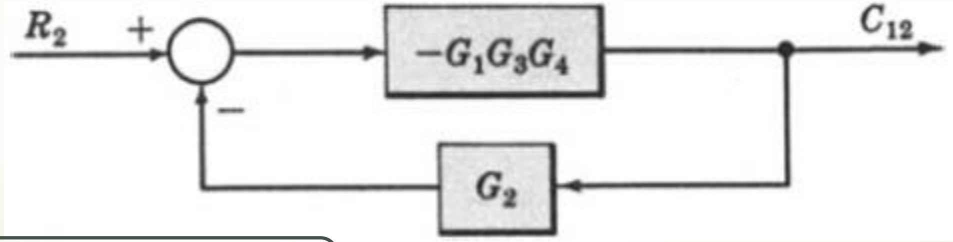
Example-17: Continue

Letting $R_2 = 0$ and combining the summing points,



Hence C_{11}, the output at C_1 due to R_1 alone, is $C_{11} = G_1 R_1/(1 - G_1G_2G_3G_4)$.

For $R_1 = 0$,



Hence $C_{12} = -G_1G_3G_4R_2/(1 - G_1G_2G_3G_4)$ is the output at C_1 due to R_2 alone.

Thus $C_1 = C_{11} + C_{12} = (G_1R_1 - G_1G_3G_4R_2)/(1 - G_1G_2G_3G_4)$

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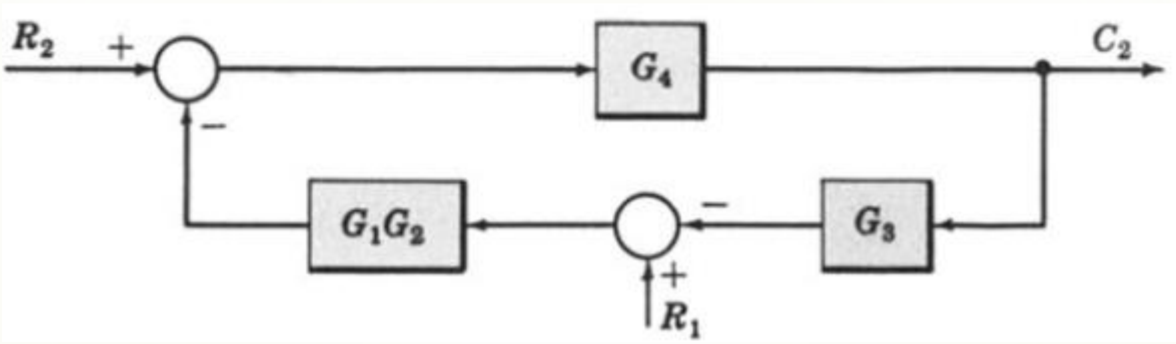
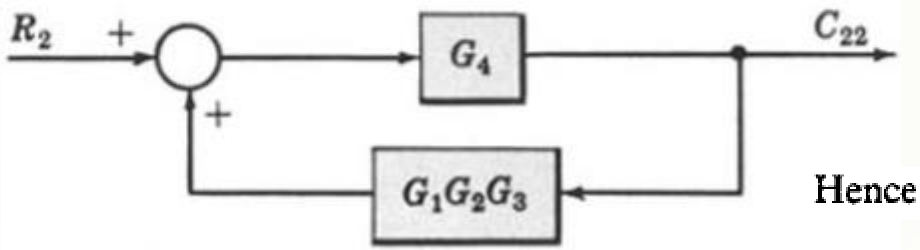
Example-17: Continue

Now we reduce the original block diagram, ignoring output C_1.

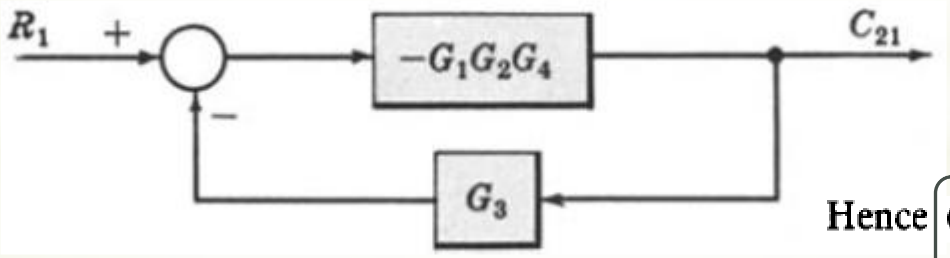


When R1 = 0,



Hence $C_{22} = G_4 R_2/(1 - G_1G_2G_3G_4)$

When R2 = 0,



Hence $C_{21} = -G_1G_2G_4 R_1/(1 - G_1G_2G_3G_4)$

Finally, $C_2 = C_{22} + C_{21} = (G_4 R_2 - G_1G_2G_4 R_1)/(1 - G_1G_2G_3G_4)$

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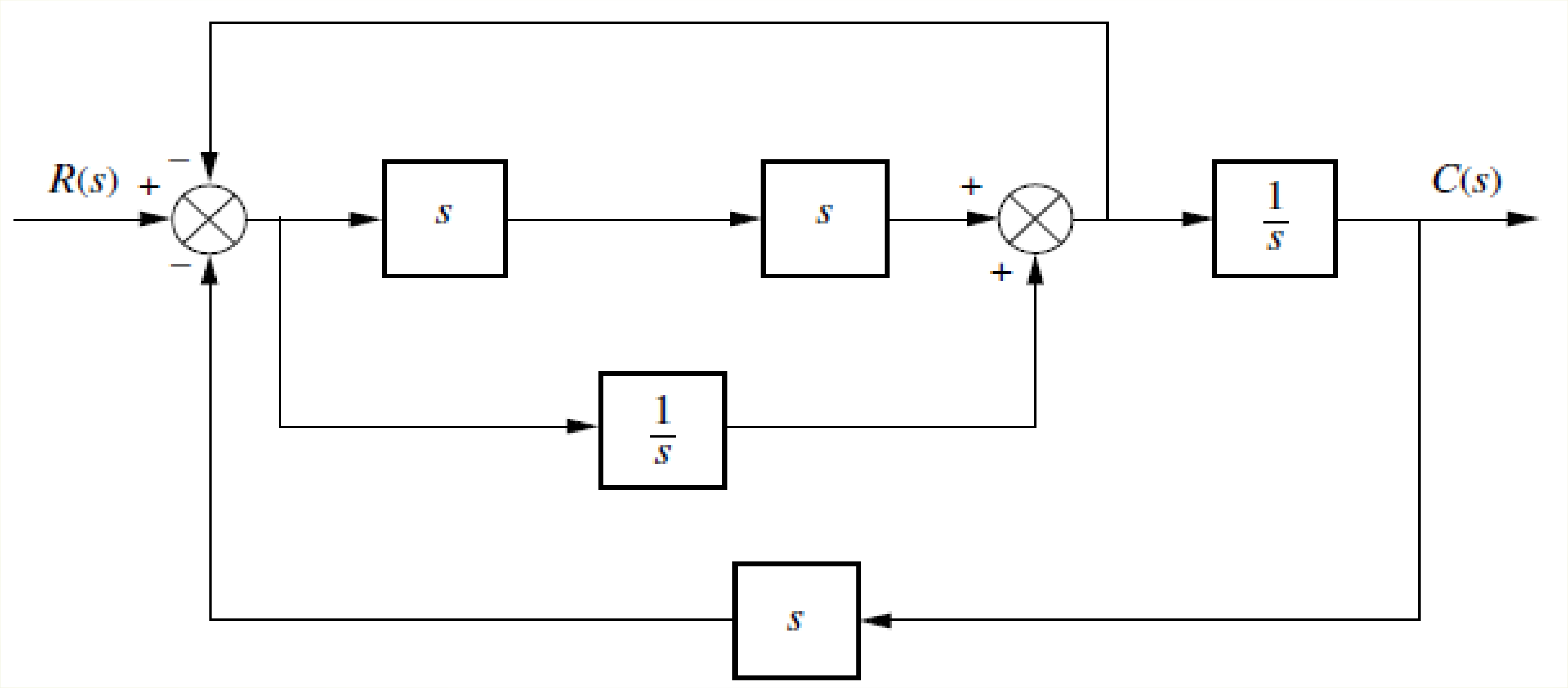
Skill Assessment Exercise:

PROBLEM: Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system



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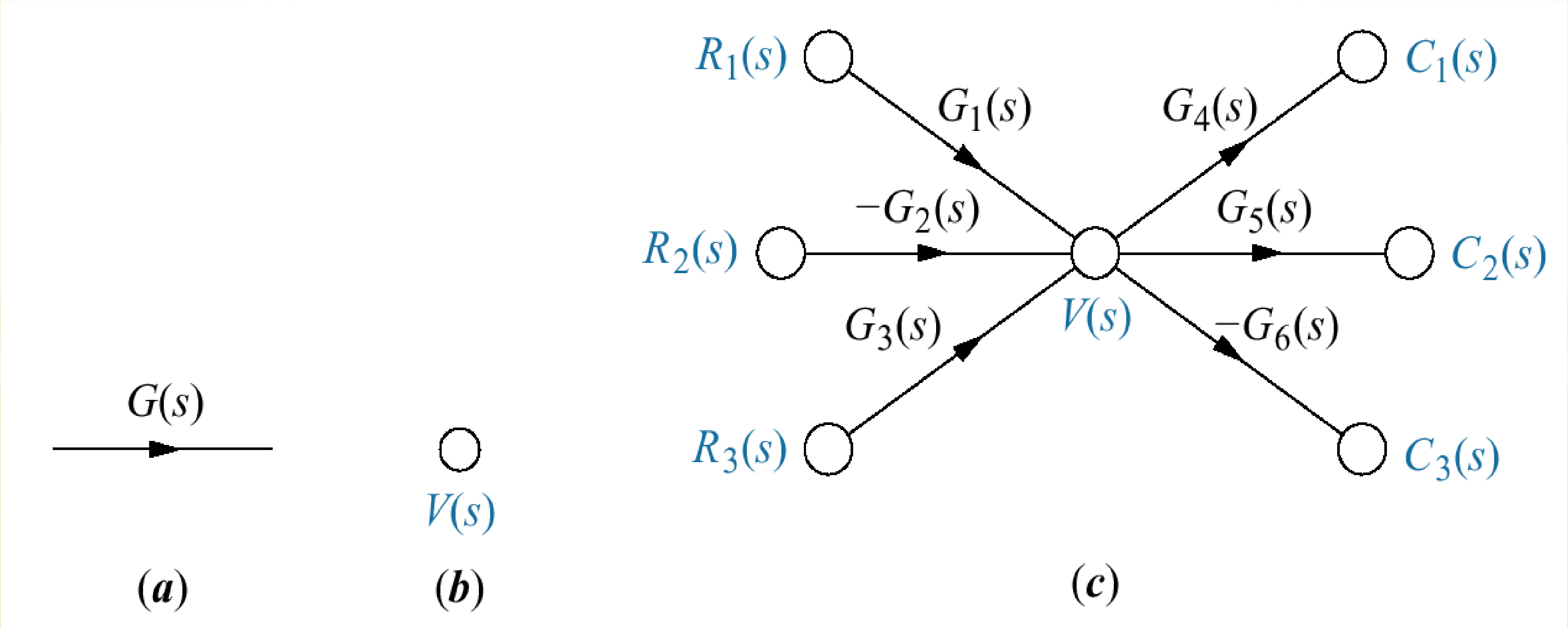
Answer of Skill Assessment Exercise:

ANSWER: $$T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

Signal-flow Graph Components

a. system;
b. signal;
c. interconnection of systems and signals



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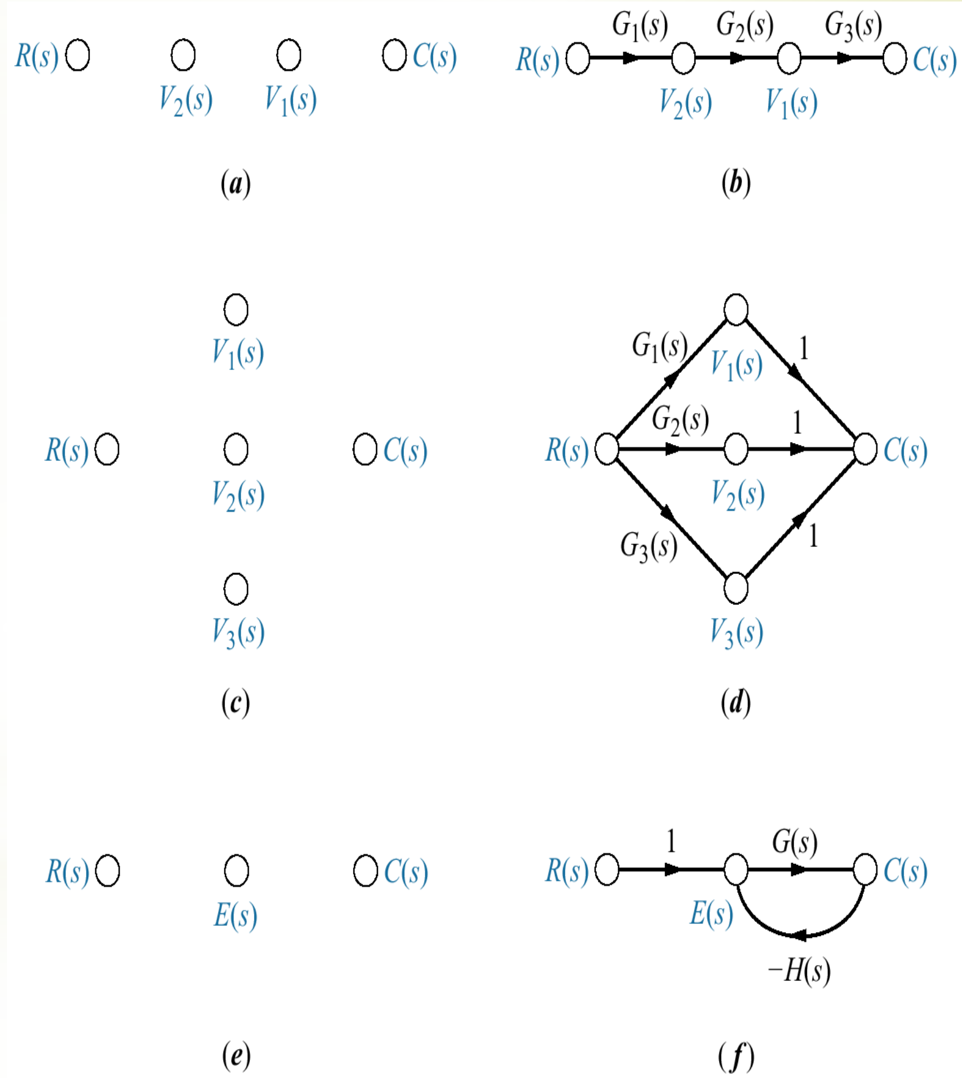
Building Signal-flow Graphs

a. Cascaded system nodes
b. Cascaded system signal-flow graph;
c. Parallel system nodes
d. Parallel system signal-flow graph;
e. Feedback system nodes
f. Feedback system signal-flow graph



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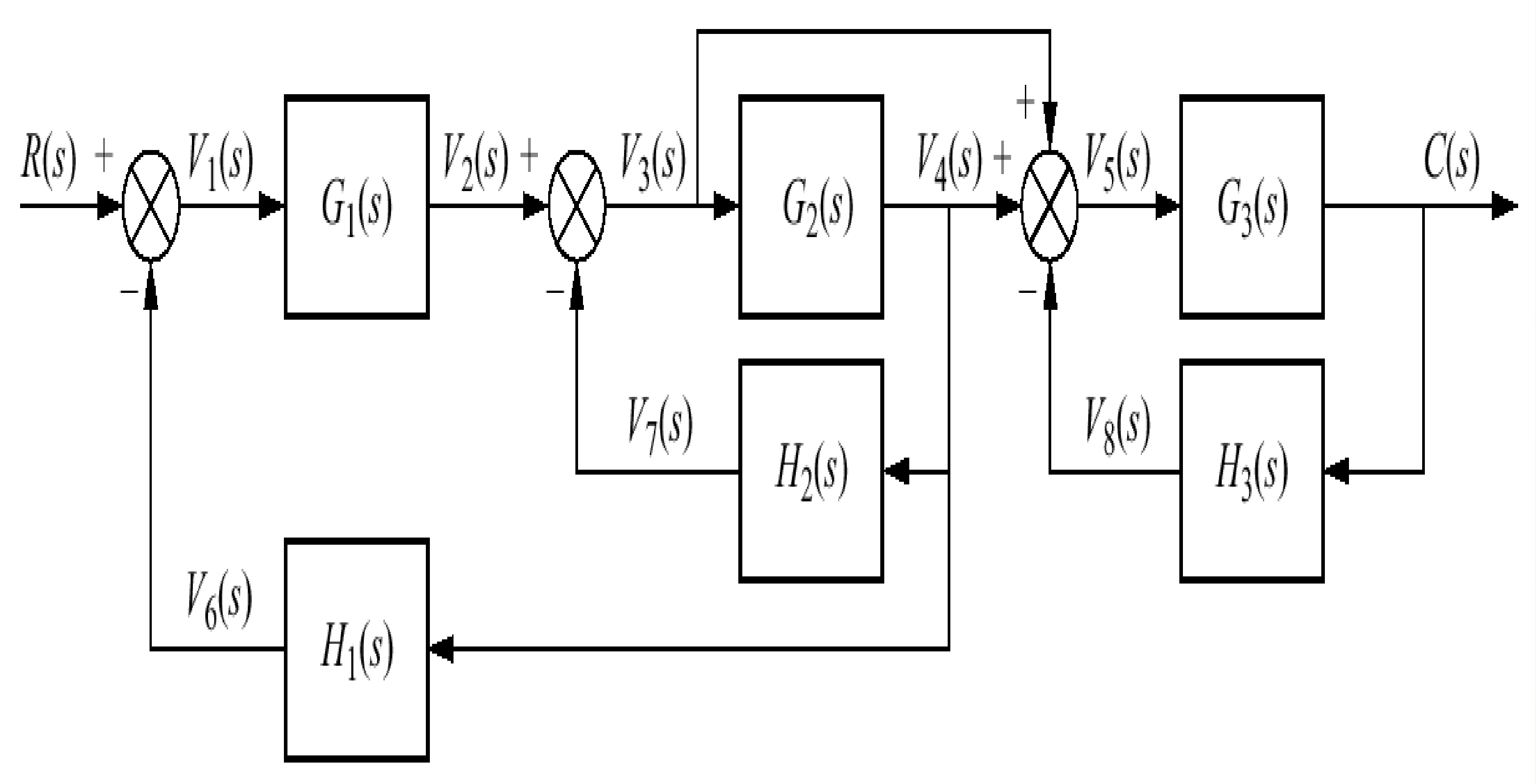
Converting a Block Diagram to a Signal-flow Graph

Problem: Convert the block diagram to a signal-flow graph.



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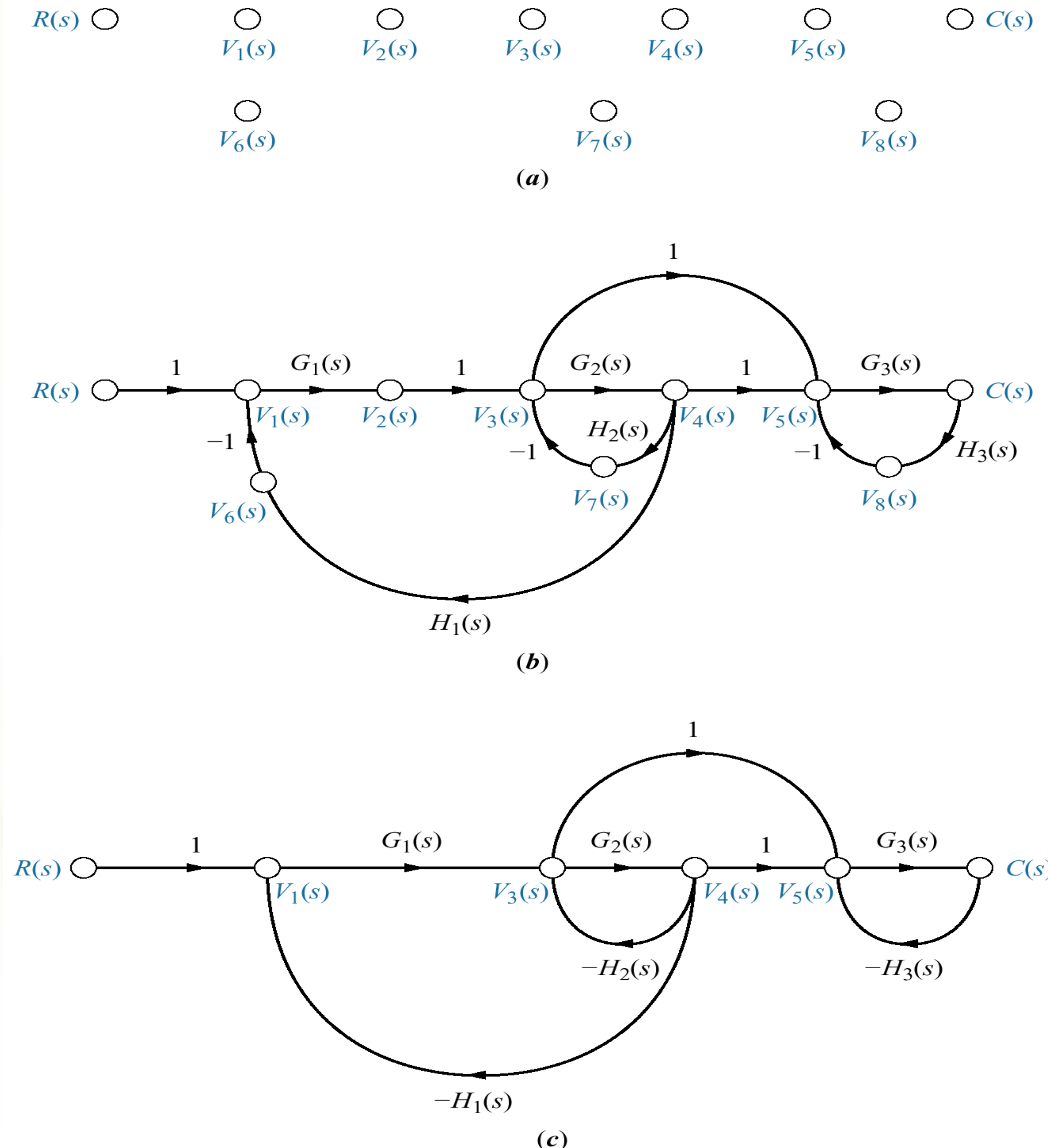
Converting a Block Diagram to a Signal-flow Graph

Signal-flow graph development:

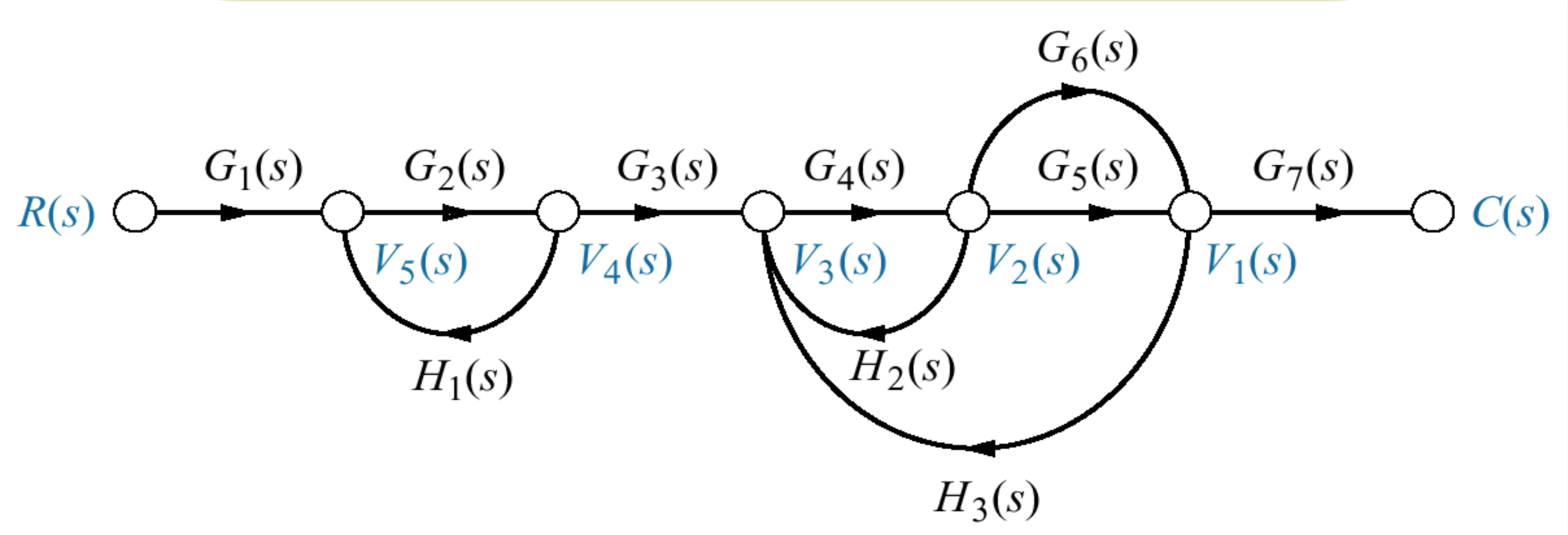
a. signal nodes;

b. signal-flow graph;

c. simplified signal-flow graph



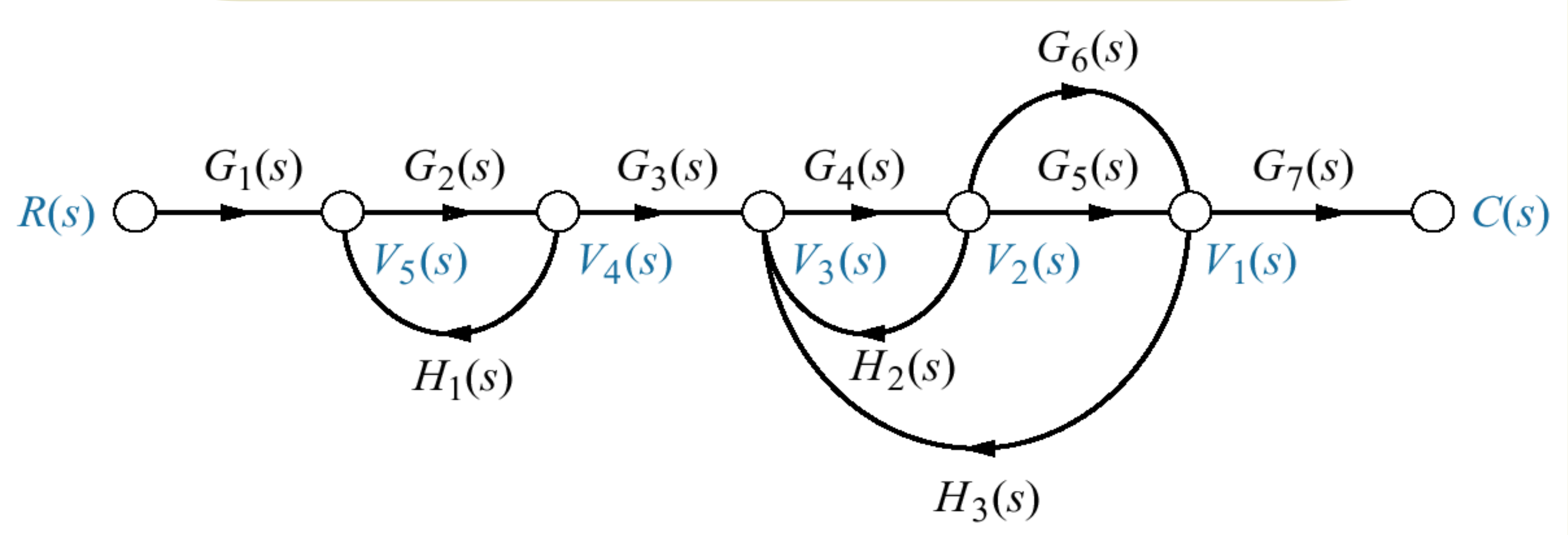
Mason's Rule - Definitions



Loop gain "L_k": The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once. **$G_2(s)H_1(s)$, $G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, $G_4(s)G_6(s)H_3(s)$**

Forward-path gain "P_k": The product of gains found by traversing a path from input node to output node in the direction of signal flow. **$G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$, $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$**

Mason's Rule - Definitions



Nontouching loops: loops that do not have any nodes in common. **$G_2(s)H_1(s)$** does not touch **$G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, and $G_4(s)G_6(s)H_3(s)$**

Nontouching-loop gain: The product of loop gains from nontouching loops taken 2, 3,4, or more at a time.

$[G_2(s)H_1(s)][G_4(s)H_2(s)]$, $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$, $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Mason's Rule

The Transfer function. C(s)/ R(s), of a system represented by a signal-flow graph is

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k P_k \Delta_k}{\Delta}$$

Where

K = number of forward paths

P_k = the k^{th} forward-path gain

Δ = 1 - $\sum$loop gains +$\sum$ nontouching-loop gains taken 2 at a time - $\sum$ nontouching-loop gains taken 3 at a time + $\sum$ nontouching-loop gains taken 4 at a time - …….

Δ_k = 1 - loop gain terms that does not touch the k*th* forward path.

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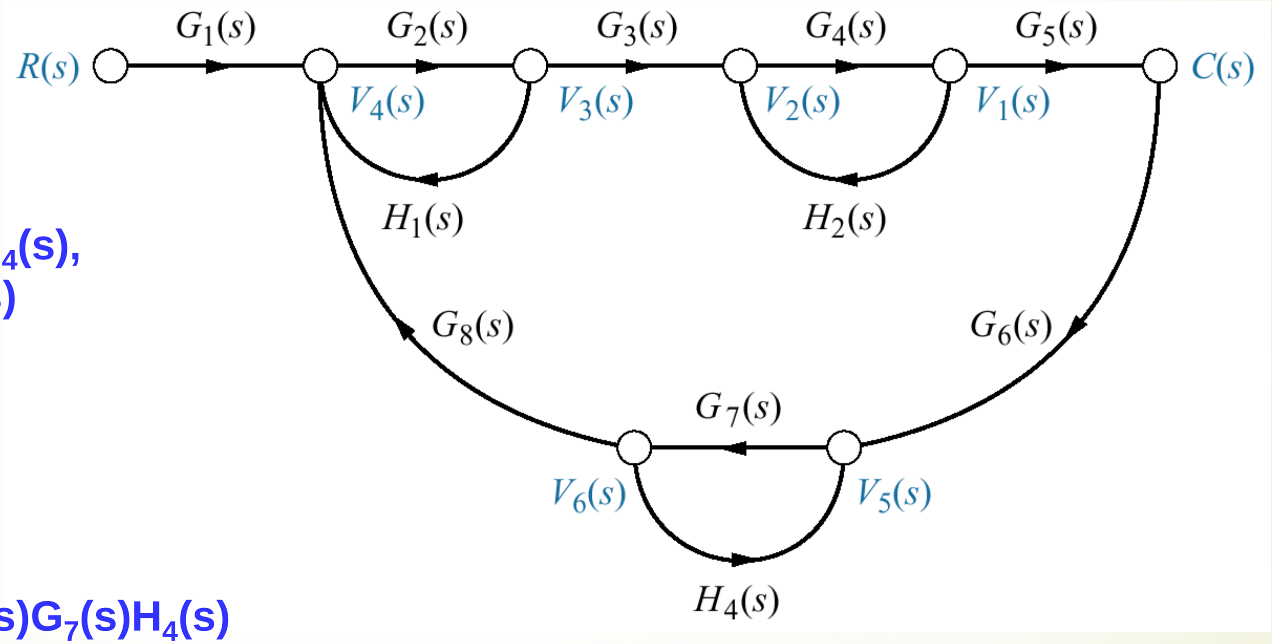
Transfer Function via Mason's Rule

Problem: Find the transfer function for the signal flow graph

Solution:



forward path

$G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$

Loop gains

$G_2(s)H_1(s)$, $G_4(s)H_2(s)$, $G_7(s)H_4(s)$, $G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)G_8(s)$

Nontouching loops

2 at a time

$G_2(s)H_1(s)G_4(s)H_2(s)$

$G_2(s)H_1(s)G_7(s)H_4(s)$

$G_4(s)H_2(s)G_7(s)H_4(s)$

3 at a time **$G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$**

Now

$$\Delta = 1-[G_2(s)H_1(s)+G_4(s)H_2(s)+G_7(s)H_4(s)+G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)G_8(s)] + [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) + G_4(s)H_2(s)G_7(s)H_4(s)] - [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

$$\Delta_1 = 1 - G7(s)H4(s)$$

$$G(s) = \frac{P_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1-G_7(s)H_4(s)]}{\Delta}$$

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Signal-Flow Graphs of State Equations

Problem: draw signal-flow graph for:

$$\dot{x}_1 = 2x_1 - 5x_2 + 3x_3 + 2r$$

$$\dot{x}_2 = -6x_1 - 2x_2 + 2x_3 + 5r$$

$$\dot{x}_3 = x_1 - 3x_2 - 4x_3 + 7r$$

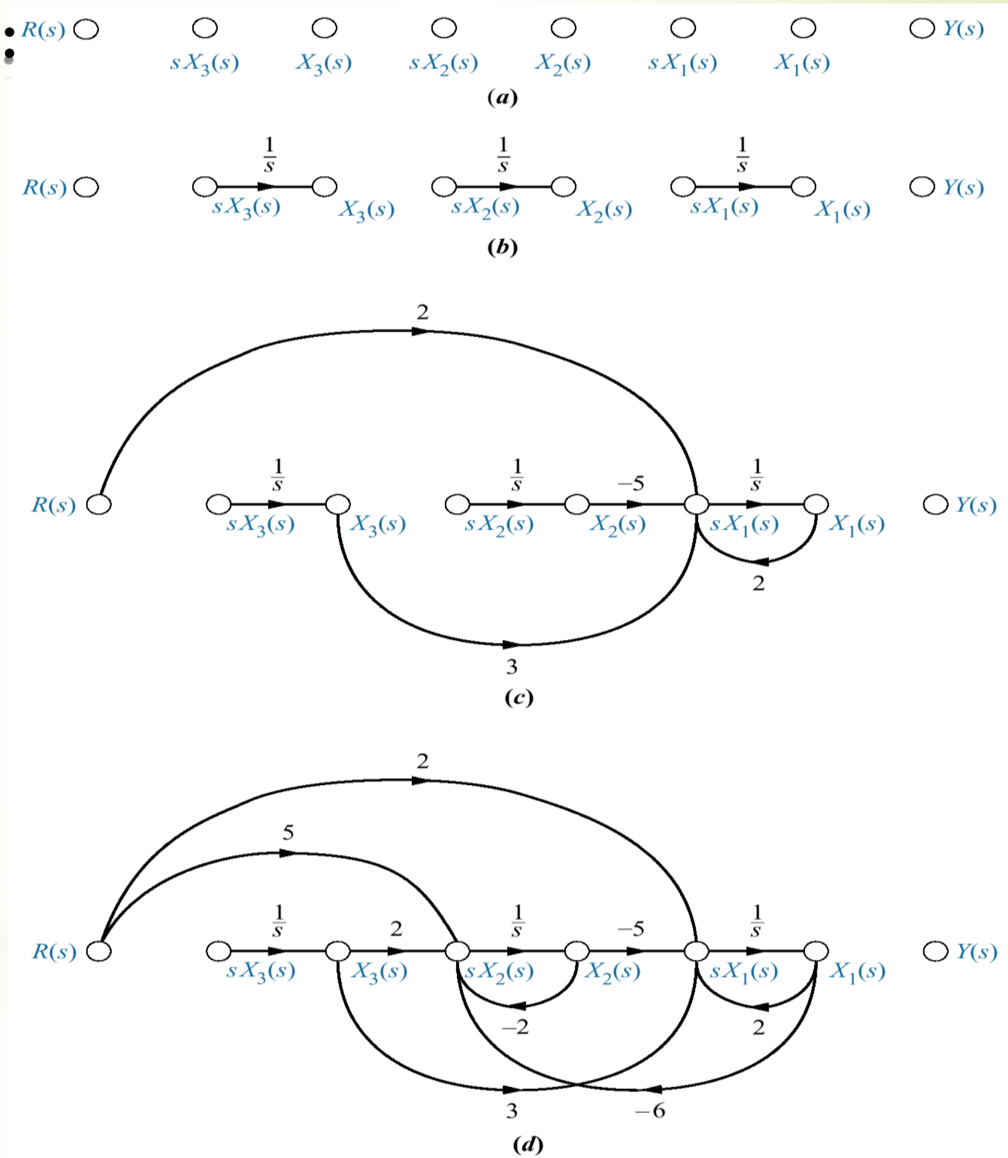
$$y = -4x_1 + 6x_2 + 9x_3$$

a. place nodes;

b. interconnect state variables and derivatives;

c. form dx1/dt ;

d. form dx2/dt



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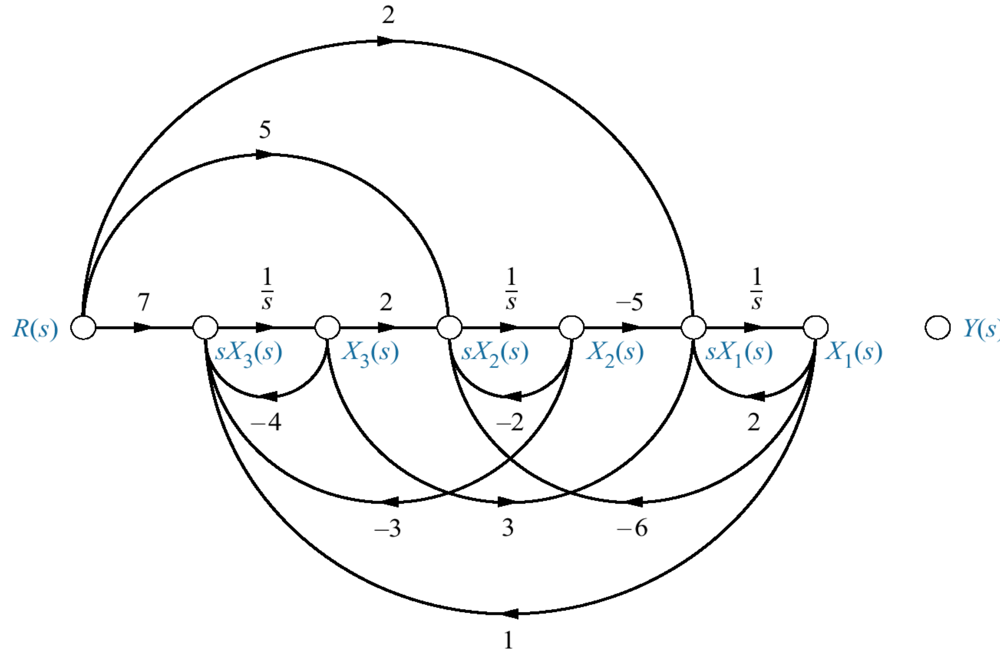
Signal-Flow Graphs of State Equations

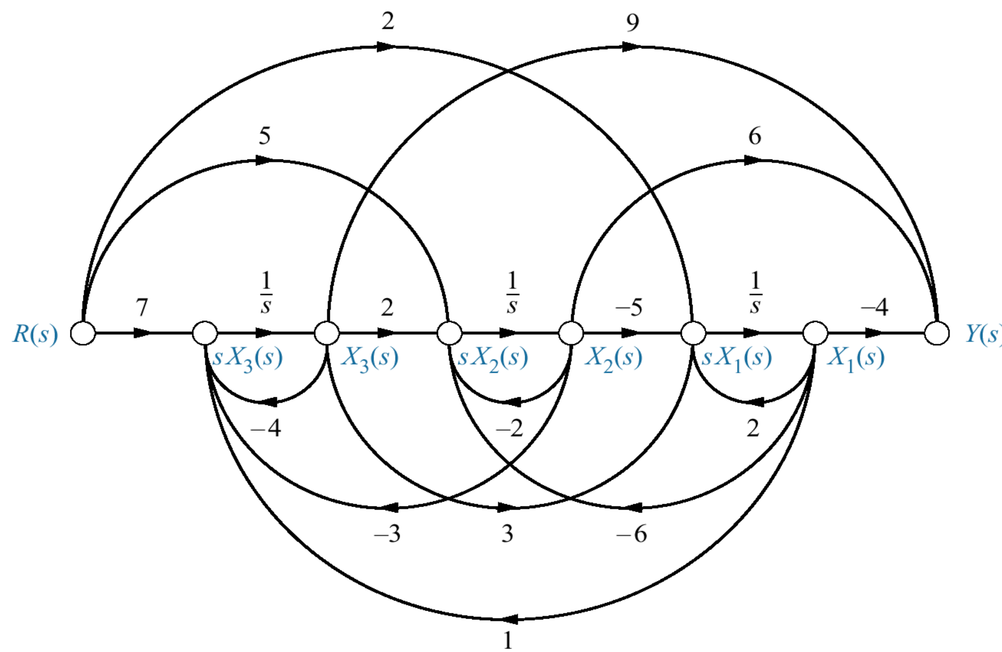
(continued)

e. form dx_3/dt;

f. form output



(e)



(f)

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Course Staff

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Thank You

For Your Attention

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